

**INSTITUTE OF ACTUARIES OF INDIA**  
**EXAMINATIONS**

**May 2025**

**CM2 - Economic Modelling (Paper B)**

**Time allowed: 1 Hours 45 Minutes**

**Total Marks: 100**

**Q.1)** An investor has shares of a local public sector bank. The current share price is INR 100, the estimated volatility for the share price is 17%. On the national exchange, there are futures and European options on these shares. The risk-free rate is 6% per annum. No dividends are paid during this time.

The investor purchases put and call options,

|             | Units | Terms | Strike Price |
|-------------|-------|-------|--------------|
| Call option | 2000  | 2.5   | 150          |
| Put option  | 10000 | 2.5   | 80           |

i) Calculate the price of the put and call options. (8)

ii) Calculate the value of the investor's portfolio at time  $t = 0$ . (2)

iii) Calculate the delta and gamma of the two options. (assume risk free rate  $\delta = 6\%$ ) (6)

**[16 Marks]**

**Q.2)** An insurance company has a portfolio of policies. Consider an aggregate claims process,  $S(t)$ , where:

- $S(t) = \sum_{i=1}^{N(t)} X_i$
- $N(t)$  = the number of claims generated by the portfolio in the time interval  $[0, t]$
- $X_i$  = the amount of the  $i$ th claim.

You have been given a single realisation of  $N(4)$  and the values of the first three claim amounts, which follow an exponential distribution with mean 1. Assume that  $c$ , the rate of premium income per unit time, is 1.05 and  $U$ , the initial surplus, is 0.6.

i) Calculate the value of the aggregate claims process at time  $t = 4$ . (2)

ii) Calculate the value of the surplus process,  $U(t)$ , at time  $t = 4$ . (4)

$N(t)$  is a Poisson process with  $\lambda = 1$ , therefore the interval of time between claims is exponentially distributed with a mean of 1. You have been given realisations of the time intervals  $t_i$  between the first three claims, where  $t_1$  is the time of the first claim and  $t_i$  (for  $i > 1$ ) is the time between claim  $i$  and the immediately preceding claim  $i - 1$ .

iii)

a) Calculate the values of  $X_1, X_2, X_3$  for each value of  $t$  from 0.01 upto 4. (3)

b) Calculate the corresponding values of  $S(t)$  and  $c$  and  $u$  for each  $t$ . (4)

c) Plot a chart showing the surplus process from time  $t = 0$  to time  $t = 4$ . (3)

iv) Calculate the minimum value of  $U$  that would avoid ruin before time  $t = 4$  in the process. (4)

The formula for the probability of ultimate ruin is  $\Psi(U)$ . Individual claim amounts are exponentially distributed with mean 1 and the premium loading factor is  $\theta$ . The formula for  $\Psi(U)$  is:

$$\Psi(U) = \frac{1}{1 + \theta} \exp \left\{ -\frac{\theta U}{1 + \theta} \right\}$$

- v) Plot a chart showing how the probability of ruin varies for values of  $\theta$  ranging from 0 to 1, when  $U = 0.5$ . (6)

**[26 Marks]**

**Q.3)** There are 100 simulations over a 10-year period with annual returns provided.

- i) Calculate the average over 10 years of TWRR p.a of the product using the simulated returns. (4)

A finance company offers an investment product. This product has 2 variants one with and one without a guaranteed return. The term is 10 years for both variants and the guarantee is 4.5%. If the initial investment is 1lac,

- ii) calculate the return under the normal variant. (4)

- iii) Calculate the return with guarantee. (6)

- iv) If the applicable current discount rate is 4%, calculate the present value of the guarantee at maturity. (6)

- v) To meet this guarantee, the company invests in 70% equity and 30% debt. The average equity return expected is 4.6% and debt return is expected to be 3% over the next 10 years. Comment on the choice of asset mix that company has made. (4)

**[24 Marks]**

**Q.4)** A general insurance company is reserving for a product liability book of business. Product liability claims arise against companies for injury/ damage occurring from defective products/ faulty manuals, etc. The incurred claims triangle is provided in the spreadsheet.

- i) Determine the reserve at the end of 2024 using the chain ladder method. (7)

Some claims can be contested in courts as well, hence they take longer to be reported and settled. However, the company has only 4 years' experience and so there may be more claims that cannot be estimated using this triangle. The chief actuary suggests applying a tail factor to all years to account for this late development. The tail factor suggested is 2.5% for 2021-22 and 5% for more recent years. The tail factor represents the claims yet to be reported/ development yet to happen in each year.

- ii) Calculate the new reserves after applying this adjustment. (5)

Instead of applying a tail factor, an alternative method that is being suggested is the Bornhuetter Ferguson method.

- iii) Calculate the reserves using this method. (6)

- iv) Comment on the results and suitability of the methods used – BCL, BF and adjustment using tail factor. (6)

**[24 Marks]**

**Q.5)** An investor makes contributions of INR 5lacs per year into a fund. You are given 5 simulations of returns over 10 years. The investor has a utility function  $U(W) = 0.00001w$ , for  $w \leq 60L$  and 85 for  $w > 60L$ .

- i) Calculate the expected fund value at the end of 10 years. (4)

- ii) Calculate the probability of returns exceeding Rs. 60L in 5 years. (2)

iii) Calculate the expected utility of the fund at the end of 10 years.

(4)

**[10 Marks]**

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